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International Journal of Heat and Mass Transfer 49 (2006) 1723–1730

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Heat and fluid flow characteristics of gases in micropipes

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Received 4 April 2005; received in revised form 15 October 2005
Available online 13 December 2005

Abstract

In this study, laminar forced convective heat transfer of a Newtonian fluid in a micropipe is analyzed by taking the viscous dissipation effect, the velocity slip and the temperature jump at the wall into account. Hydrodynamically and thermally fully developed flow case is examined. Two different thermal boundary conditions are considered: the constant heat flux (CHF) and the constant wall temperature (CWT). Either wall heating (the fluid is heated) case or wall cooling (the fluid is cooled) case is examined. The Nusselt numbers are analytically determined as a function of the Brinkman number and the Knudsen number. Different definitions of the Brinkman number based on the definition of the dimensionless temperature are discussed. It is disclosed that for the cases studied here, singularities for the Brinkman number-dependence of the Nusselt number are observed and they are discussed in view of the energy balance.
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Keywords: Microscale; MEMS; Flow physics; Viscous dissipation; Temperature jump; Velocity slip

1. Introduction

Microelectromechanical systems (MEMS) has gained a great deal of interest in recent years. Such small devices typically have characteristic size ranging from 1 mm down and to 1 micron, and may include sensors, actuators, motors, pumps, turbines, gears, ducts and valves. Microdevices often involve mass, momentum and energy transport. Modeling gas and liquid flows through MEMS may necessitate including slip, rarefaction, compressibility, intermolecular forces and other unconventional effects [1].

It is shown that fluid flow and heat transfer at microscale differ greatly from those at macroscale. At macroscale, classical conservation equations are successfully coupled with the corresponding wall boundary conditions, usual no-slip for the hydrodynamic boundary condition and no-temperature-jump for the thermal boundary condition. These two boundary conditions are valid only if the fluid flow adjacent to the surface is in thermal equilibrium. However, they are not valid for gas flow at microscale. For this case, the gas no longer reaches the velocity or the temperature of the surface and therefore a slip condition for the velocity and a jump condition for the temperature should be adopted.

The Knudsen number, Kn is the ratio of the gas mean free path, λ, to the characteristic dimension in the flow field, D, and, it determines the degree of rarefaction and the degree of the validity of the continuum approach. As Kn increases, rarefaction become more important, and eventually the continuum approach breaks down. The following regimes are defined based on the value of Kn [2].

(i) Continuum flow (ordinary density levels) $Kn \leqslant 0.001$.
(ii) Slip-flow regime (slightly rarefied) $0.001 \leqslant Kn \leqslant 0.1$.
(iii) Transition regime (moderately rarefied) $0.1 \leqslant Kn \leqslant 10$.
(iv) Free-molecule flow (highly rarefied) $10 \leqslant Kn \leqslant \infty$.

Viscous dissipation is another parameter that should be taken into consideration at microscale. It changes temperature distributions by playing a role like an energy source induced by the shear stress, which, in the following, affects heat transfer rates. The merit of the effect of the viscous dissipation depends on whether the pipe is being cooled or heated.

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0017-9310/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2005.10.020

Nomenclature

A	cross-sectional area [m^2]
Br	Brinkman number, Eq. (10)
Br_q	modified Brinkman number, Eq. (13)
c_p	specific heat at constant pressure
D	diameter of the pipe [m]
F	tangential momentum accommodation coefficient
F_t	thermal accommodation coefficient
h	convective heat transfer coefficient [W/m^2 K]
k	thermal conductivity [W/m K]
Kn	Knudsen number
Nu	Nusselt number
Pr	Prandtl number
q_w	wall heat flux [W/m^2]
R	dimensionless radial coordinate
r	radial coordinate [m]
r_0	radius of the pipe
T	temperature [K]
u	velocity [m/s]
z	axial direction [m]
Greek symbols	
α	thermal diffusivity [m^2/s]
γ	specific heat ratio
λ	molecular mean free path
μ	dynamic viscosity [Pa s]
ρ	density [kg/m^3]
υ	kinematic viscosity [m^2/s]
θ	dimensionless temperature, Eq. (8)
θ_q	dimensionless temperature modified, Eq. (13)
θ^*	dimensionless temperature, Eq. (16)
θ_q^*	dimensionless temperature modified, Eq. (17)
Subscripts	
c	centerline
m	mean
s	fluid properties at the wall, singularity value
w	wall

There is a scarcity of experimental data and theoretical analysis available in the existing literature, many of which are contradictory and conflicting, yielding different correlations with opposite characteristics. Therefore, the mechanisms of flow and heat transfer in microchannels are still not understood clearly.

Readers are referred to see the following recent excellent reviews related to transport phenomena in microchannels. Ho and Tai [3] summarized discrepancies between microchannel flow behavior and macroscale Stokes flow theory. Palm [4], Sobhan and Garimella [5] and Obot [6] reviewed the experimental results in the existing literature for the convective heat transfer in microchannels. Rostami et al. [7,8] presented reviews for flow and heat transfer of liquids and gases in microchannels. Gad-el-Hak [1] broadly surveyed available methodologies to model and compute transport phenomena within microdevices. Guo and Li [9,10] reviewed and discussed the size effects on microscale single-phase fluid flow and heat transfer. In a recent study, Morini [11] presents an excellent review of the experimental data for the convective heat transfer in microchannels in the existing literature. He critically analyzed and compared the results in terms of the friction factor, laminar-to-turbulent transition and the Nusselt number.

Gravesen et al. [12] explained deviations at the microscale from the macroscale in terms of wall slip and compressibility phenomena in microchannels. Gaseous flow in two-dimensional (2-D) micromachined channels with a Cartesian geometry for various Knudsen numbers was studied by Harley et al. [13]. Barron et al. [14,15] extended the Graetz problem to slip- flow and developed simplified relationships to describe the effect of slip-flow on the convection heat transfer coefficient. Ameel et al. [16] analytically treated the problem of laminar gas flow in microtubes with a constant heat flux boundary condition at the wall assuming a slip flow hydrodynamic condition and a temperature jump thermal condition at the wall. They disclosed that the fully developed Nusselt number decreased with Knudsen number. Tso and Mahulikar [17–19] studied the effect of the Brinkman number on convective heat transfer and flow transition in microchannels. Tunc and Bayazitoglu [20] studied steady laminar hydrodynamically developed flow in microtubes with uniform temperature and uniform heat flux boundary conditions using the integral technique. Toh et al. [21] numerically investigated three-dimensional fluid flow and heat transfer phenomena inside heated microchannels using a finite volume method. Xu et al. [22] theoretically analyzed and examined the effects of viscous dissipation in microchannel flows. They suggested a criterion to draw the limit of the significance of the viscous dissipation effects. Koo and Kleinstreuer [23,24] investigated the effects of viscous dissipation on the temperature field and ultimately on the friction factor using dimensional analysis and experimentally validated computer simulations. Hsieh et al. [25] presented an experimental and theoretical study of low Reynolds number flow of nitrogen in a microchannel. They concluded that using slip boundary conditions, one could well predict the mass flow rate as well as inlet/exit pressure drop and friction factor constant ratio for a three-dimensional physical system. In a recent study, Aydın [26] investigated the effect of the viscous dissipation on the heat transfer for a hydrodynamically and thermally fully developed flow in a pipe.

The purpose of the present study is to theoretically investigate the gas flow in a micropipe. Both the constant wall temperature (CWT) and the constant heat flux (CHF) thermal boundary conditions are applied at the wall. Either wall heating (the fluid is heated) case or wall cooling (the fluid is cooled) case is considered. The combined effects of the Brinkman number and the Knudsen number on the temperature profile and the Nusselt number are determined.

2. Analysis

In this case, the flow is considered to be fully developed both hydrodynamically and thermally. Steady, laminar flow having constant properties (i.e. The thermal conductivity and the thermal diffusivity of the fluid are considered to be independent of temperature) is considered. The axial heat conduction in the fluid and in the wall is assumed to be negligible.

In this study, the usual continuum approach is coupled with the two main characteristics of the microscale phenomena, the velocity slip and the temperature jump, following the Refs. [16,20].

Velocity slip is defined as [16]:

$$u_s = -\frac{2-F}{F}\lambda \left.\frac{\partial u}{\partial r}\right|_{r=r_0} \tag{1}$$

where u_s is the slip velocity, λ the molecular mean free path, and F is the tangential momentum accommodation coefficient, and the temperature jump is defined as [27]:

$$T_s - T_w = -\frac{2-F_t}{F_t}\frac{2\gamma}{\gamma+1}\frac{\lambda}{Pr}\left.\frac{\partial T}{\partial r}\right|_{r=r_0} \tag{2}$$

where T_s is the temperature of the gas at the wall, T_w the wall temperature , and F_t is the thermal accommodation coefficient. For the rest of the analysis, F and F_t will be shown by F and assumed to be 1.

The fully developed velocity profile taking the slip flow condition at the wall into consideration is derived from the momentum equation as

$$u = \frac{2u_m(1-(r/r_0)^2+4Kn)}{(1+8Kn)} \tag{3}$$

where u_m is the mean velocity and Kn is the Knudsen number, $Kn = \lambda/D$.

The conservation of energy including the effect of the viscous dissipation requires

$$u\frac{\partial T}{\partial z} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial r}\right)^2 \tag{4}$$

where the second term in the right hand side is the viscous dissipation term.

Due to axisymmetry at the center, the thermal boundary condition at $r = 0$ can be written as

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \tag{5}$$

Thermal boundary conditions of constant wall heat flux (CHF) and constant wall temperature (CWT) at wall are considered and they are separately treated in the following:

2.1. CHF case

The constant heat flux at wall states that

$$k\left.\frac{\partial T}{\partial r}\right|_{r=r_0} = q_w \tag{6}$$

where q_w is positive when its direction is to the fluid (the hot wall), otherwise it is negative (the cold wall).

For the uniform wall heat flux case, the first term in the left-side of Eq. (4) is

$$\frac{\partial T}{\partial z} = \frac{dT_w}{dz} = \frac{dT_s}{dz} \tag{7}$$

By introducing the following non-dimensional quantities:

$$R = \frac{r}{r_0} \quad \theta = \frac{T_s - T}{T_s - T_c} \tag{8}$$

Eq. (4) can be written as

$$\frac{d}{dR}\left(R\frac{d\theta}{dR}\right) = a\frac{2(R-R^3+4KnR)}{(1+8Kn)} + 16Br\frac{R^3}{(1+8Kn)^2} \tag{9}$$

where $a = -\frac{u_m r_0^2}{\alpha(T_s - T_c)}\frac{dT_s}{dz}$ and Br is the Brinkman number given as

$$Br = \frac{\mu u_m^2}{k(T_s - T_c)} \tag{10}$$

For the solution of the dimensionless energy transport equation given in Eq. (9), the dimensionless boundary conditions are given as follows:

$$\begin{aligned} &\theta = 1 \quad \left.\frac{\partial\theta}{\partial R}\right|_{R=0} = 0 \quad \text{at } R = 0 \\ &\theta = 0 \quad \text{at } R = 1 \end{aligned} \tag{11}$$

The solution of Eq. (9) under the thermal boundary conditions given in Eq. (11) is

$$\begin{aligned}\theta(R) &= \frac{T_s - T}{T_s - T_c} \\ &= \frac{1}{3+16Kn}\left[3 + R^4 - 4R^2 + 16Kn(1-R^2)\frac{4Br}{(1+8Kn)^2}\right. \\ &\quad \left.\times\left(R^4 - R^2 + 4(R^4 - R^2)\right)\right]\end{aligned} \tag{12}$$

As it is usual in the existing literature, we can also use the modified Brinkman number which is

$$Br_q = \frac{\mu u_m^2}{Dq_w} \tag{13}$$

In terms of the modified Brinkman number (based on the wall heat flux) given above, the temperature distribution is obtained as

$$\theta_q = \frac{T - T_s}{\frac{q_w r_0}{k}} = \left[\frac{4}{1+8Kn} + \frac{32Br_q}{(1+8Kn)^3}\right]\left[-\frac{3}{16} + \frac{R^2}{4} - \frac{R^4}{16} + Kn(R^2-1)\right] - \frac{2Br_q}{(1+8Kn)^2}(R^4-1) \tag{14}$$

Eqs. (12) and (14) which are in terms of T_s (note again T_s is the temperature of the fluid at the wall) can be transformed into the equations in terms of T_w, the wall temperature using the following conversion formula:

$$\frac{T_s - T_w}{\frac{q_w r_0}{k}} = -\frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}$$
$$\frac{T_s - T_w}{T_s - T_c} = \frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}\left.\frac{\partial\theta}{\partial R}\right|_{R=1} \tag{15}$$

Then Eqs. (12) and (14) become, respectively,

$$\theta^*(R) = \frac{T_w - T}{T_s - T_c} = \frac{1}{3+16Kn}\left[3 + R^4 - 4R^2 + 16Kn(1-R^2) + \frac{4Br}{(1+8Kn)^2}\left(R^4 - R^2 + 4(R^4 - R^2)\right) - \frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}\left[-4 - 32Kn + Br\frac{8+32Kn}{(1+8Kn)^2}\right]\right] \tag{16}$$

and

$$\theta_q^* = \frac{T - T_w}{\frac{q_w r_0}{k}} = \left[\frac{4}{1+8Kn} + \frac{32Br_q}{(1+8Kn)^3}\right]\left[-\frac{3}{16} + \frac{R^2}{4} - \frac{R^4}{16} + Kn(R^2-1)\right] - \frac{2Br_q}{(1+8Kn)^2}(R^4-1) - \frac{4\gamma}{\gamma+1}\frac{Kn}{Pr} \tag{17}$$

In fully developed flow, it is usual to utilize the mean fluid temperature, T_m, rather than the center line temperature when defining the Nusselt number. This mean or bulk temperature is given by

$$T_m = \frac{\int \rho u T \, dA}{\int \rho u \, dA} \tag{18}$$

The dimensionless mean temperature in terms of Br is obtained as

$$\theta_m^* = \frac{T_w - T_m}{T_c - T_s} = \frac{11 - 4Br(1+4Kn) + 216Kn + 1408Kn^2 + 3072Kn^3}{6(1+8Kn)^2(3+16Kn)} - \frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}\left[-4 - 32Kn + Br\frac{8+32Kn}{(1+8Kn)^2}\right] \tag{19}$$

while, in terms of Br_q, it is derived as

$$\theta_{q,m}^* = \frac{T_m - T_w}{\frac{q_w r_0}{k}} = -\frac{1}{4}\left(1 + \frac{16\gamma}{\gamma+1}\frac{Kn}{Pr}\right) - \frac{Br_q}{3(1+8Kn)^4} - \frac{Br}{(1+8Kn)^3} - \frac{1+16Br}{24(1+8Kn)^2} - \frac{1}{6(1+8Kn)} \tag{20}$$

2.2. CWT case

When the constant temperature is considered, since $\frac{dT_s}{dz} = 0$, the first term in the left-side of Eq. (4) is

$$\frac{\partial T}{\partial z} = \left(\frac{T - T_s}{T_c - T_s}\right)\frac{dT_c}{dz} \tag{21}$$

Substituting this result into Eq. (4) and introducing the dimensionless quantities given in Eq. (8) give the following dimensionless equation for the CWT case:

$$\frac{d}{dR}\left(R\frac{d\theta}{dR}\right) = a\frac{2(R - R^3 + 4KnR)}{(1+8Kn)}\theta + 16Br\frac{R^3}{(1+8Kn)^2} \tag{22}$$

where $a = -\frac{u_m r_0^2}{\alpha(T_s - T_c)}\frac{dT_c}{dz}$ and Br is the Brinkman number. The boundary conditions given in Eq. (11) are also valid for this case. Actually, no simple closed form solution can be obtained for this equation. However, the variation of θ can be quite easily obtained to any required degree of accuracy by using an iterative procedure [28]. The temperature profile for the CHF case is used as the first approximation and Eq. (22) is then integrated to obtain θ. This iterative procedure is repeated until an acceptable convergence is obtained.

The forced convective heat transfer coefficient is given as follows:

$$h = \frac{k\left.\frac{\partial T}{\partial r}\right|_{r=r_0}}{T_w - T_m} \tag{23}$$

which is obtained from the following Nusselt number expressions based on θ^* and θ_q^*, respectively:

$$Nu = \frac{hD}{k} = -\frac{2\left.\frac{\partial\theta^*}{\partial R}\right|_{R=1}}{\theta_m^*} \tag{24}$$

$$Nu = \frac{hD}{k} = -\frac{2}{\theta_{q,m}^*} \tag{25}$$

After performing necessary substitutions, we obtain:

$$Nu = \frac{2\left[4 + 32Kn - Br\frac{8+32Kn}{(1+8Kn)^2}\right]}{\left[\frac{11 - 4Br(1+4Kn) + 216Kn + 1408Kn^2 + 3072Kn^3}{6(1+8Kn)^2(3+16Kn)} - \frac{4\gamma}{\gamma+1}\frac{Kn}{Pr}\left[-4 - 32Kn + Br\frac{8+32Kn}{(1+8Kn)^2}\right]\right]} \tag{26}$$

In terms of the modified Brinkman number, Br_q,

$$Nu = \frac{2}{\frac{1}{4}\left(1 + \frac{16\gamma}{\gamma+1}\frac{Kn}{Pr}\right) + \frac{Br_q}{3(1+8Kn)^4} + \frac{Br_q}{(1+8Kn)^3} + \frac{1+16Br_q}{24(1+8Kn)^2} + \frac{1}{6(1+8Kn)}} \tag{27}$$

3. Results and discussion

In this study, we investigate the interactive effects of the Brinkman number and Knudsen number for both hydrodynamically and thermally fully developed flow in a micropipe. Note that $Kn = 0$ represents the macroscale case, while $Kn > 0$ holds for the microscale case, and $Br = 0$ or $Br_q = 0$ represents the case without the effect of the viscous dissipation. As stated earlier, two different thermal boundary conditions at the pipe wall have been examined in this study, namely constant heat flux (CHF) and constant wall temperature (CWT). They are treated separately in the following.

At first, we validated our analysis by comparing some limiting results with those available in the existing literature, mainly by those of Ameel et al. [16] and Tunc and Bayazitoglu [20], and note for the CHF case we obtain exact solutions. Comparison of our results with those in terms of the Nusselt numbers for different Knudsen numbers showed an excellent agreement (Table 1). Note that Ameel et al. [16] did not consider the viscous dissipation effect, and Tunc and Bayazitoglu's study [20] covered the Brinkman number range of $-0.01 \leqslant Br \leqslant 0.01$.

For the CHF condition, Fig. 1 depicts the variation of the Nusselt number with the Knudsen number for different Brinkman numbers. For $Br = 0$, an increase at Kn decreases Nu due to the temperature jump at the wall. Viscous dissipation, as an energy source, severely distorts the temperature profile. Remember positive values of Br correspond to wall heating (heat is being supplied across the walls into the fluid) case ($T_w > T_c$), while the opposite is true for negative values of Br. In the absence of viscous dissipation the solution is independent of whether there is wall heating or cooling. However, viscous dissipation always contributes to internal heating of the fluid, hence the solution will differ according to the process taking place. Nu decreases with increasing Br for the hot wall (i.e. the wall heating case). For this case, the wall temperature is greater than that of the bulk fluid. Viscous dissipation increases the bulk fluid temperature especially near the wall since the highest shear rate occurs in this region. Hence, it decreases the temperature difference between the wall and the bulk fluid, which is the main driving mechanism for the heat transfer from wall to fluid. However, for the cold wall (i.e. the wall cooling case), the viscous dissipation increases the temperature differences between the wall and the bulk fluid by increasing the fluid temperature more. As seen from Fig. 1, increasing Br in the negative direction increases Nu. This situation is an indication of the aiding effect of the viscous dissipation on heat transfer for the wall cooling case. As seen from Fig. 1, the behavior of Nu versus Kn for lower values of the Brinkman number, either in the case of wall heating ($Br = 0.01$) or in the case of the wall cooling ($Br = -0.01$) is very similar to that of $Br = 0$. For the wall cooling case, at $Br = -0.1$, the decreasing effect of Kn on Nu intensifies a lot. For the wall cooling case, at $Br = 0.1$, an interesting situation is observed. Increasing Kn increases Nu up to $Kn \cong 0.01$ where a maximum occurs. Beyond this value, Kn has a decreasing effect on Nu.

Table 1
The fully developed Nusselt number values, $Br_q = 0.0$ for the CHF case

Kn	Present	Ameel and coworkers [15]	Tunc and Bayazitoglu [20]
0.00	4.364	4.364	4.3627
0.02	4.071	4.071	4.0701
0.04	3.749	3.749	3.7483
0.06	3.439	3.439	3.4383
0.08	3.156	3.156	3.1554
0.10	2.904	2.904	2.9035
0.12	2.681	2.681	2.6813

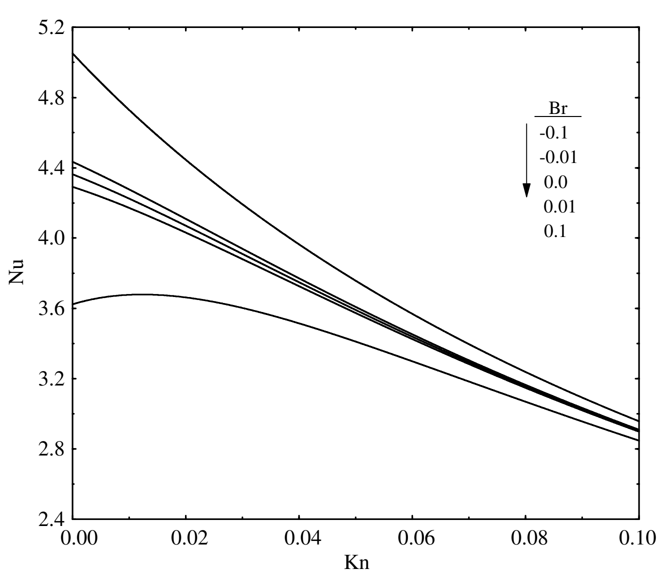

Fig. 1. The variation of the Nusselt number with the Knudsen number at different values of the Brinkman number for the CHF case.

The standard way of making temperature dimensionless based on Eq. (8) is not appropriate for the situation of imposed heat flux because the temperature scale $\Delta T = T_s - T_c$ varies with relevant parameters and may cause to a misinterpretation of the corresponding variation of T. In fact, for a given q_w, ΔT is unknown of the problem and it is more convenient to define a fixed temperature scale that we take as $q_w R/k$. Fig. 2 shows the variation of the Nusselt number with the Knudsen number for different values of the modified Brinkman number, Br_q. The behavior of Nu versus Kn for different Br_q is very similar to for different Br (Fig. 1). As expected, increasing dissipation increases the bulk temperature of the fluid due to internal heating of the fluid. For the wall heating case, this increase in the fluid temperature decreases the temperature difference between the wall and the bulk fluid, which is followed with a decrease in heat transfer. When wall cooling is applied, due to the internal heating effect of the viscous dissipation on the fluid temperature profile, temperature difference is increased with the increasing Br_q.

For the CWT case, Fig. 3 illustrates Nu versus Kn for different values of Br. Similar behaviors have been

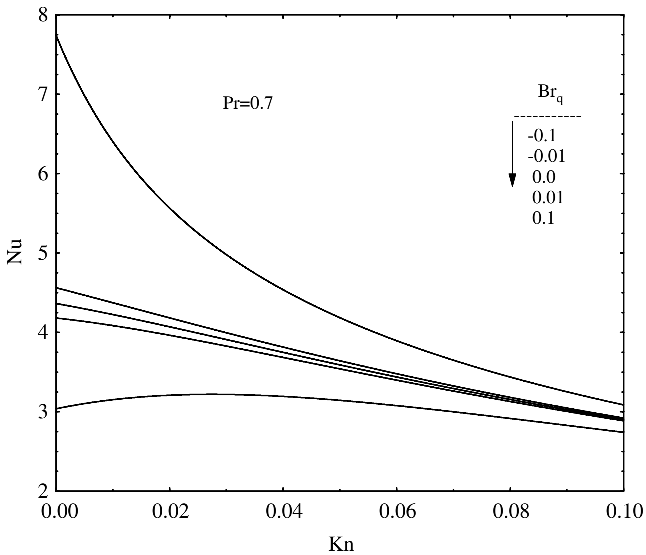

Fig. 2. The variation of the Nusselt number with the Knudsen number at different values of the modified Brinkman number for the CHF case.

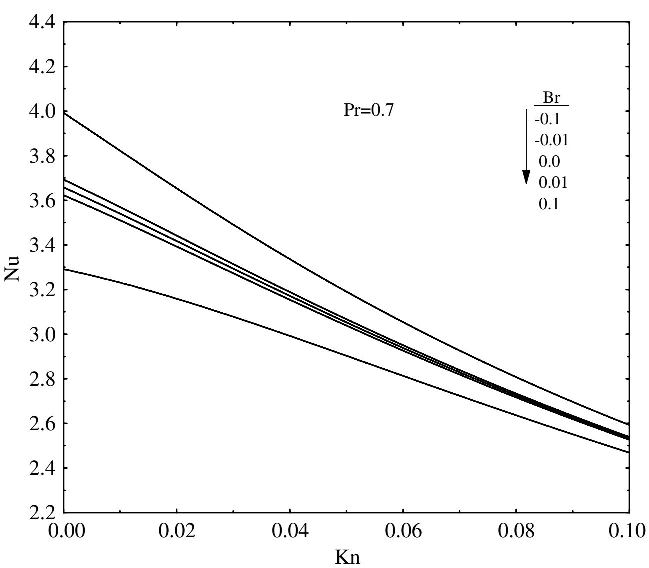

Fig. 3. The variation of the Nusselt number with the Knudsen number at different values of the Brinkman number for the CWT case.

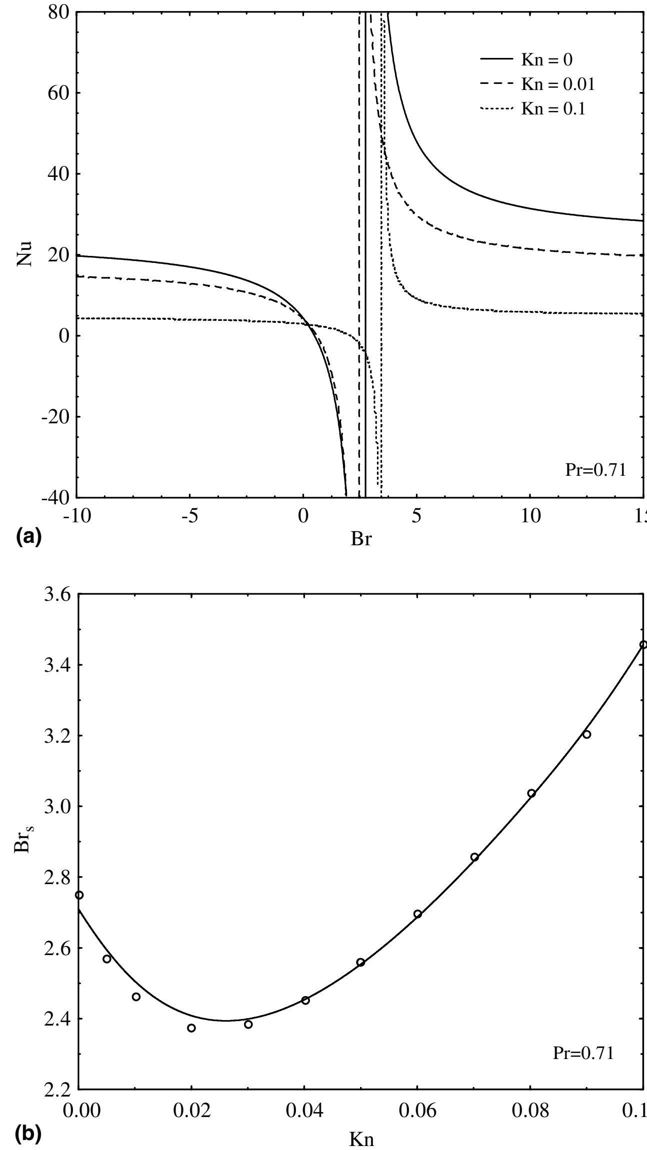

Fig. 4. (a) The influence of Br on Nu at various Kn for the CHF case. (b) The variation of Br_s with Knudsen number for the CHF case.

observed to those obtained for the CHF case. As seen, for the same value of Br, Nu values are found to be lower than those for the CHF case.

Here, as an original attempt, we extend the Brinkman number range for the above cases considered. Fig. 4a shows the influence of Br on Nu for various Kn. As shown, a singularity is observed at Br for each Kn. Actually, this is an expected result, when Eq. (26) is closely examined. For the wall heating case, at $Kn = 0$, with the increasing value of Br, Nu decreases in the range of $0 < Br < 11/4$. This is because the temperature difference which drives the heat transfer decreases. At $Br = 11/4$, the heat supplied by the wall into the fluid is balanced with the internal heat generation due to the viscous heating. For $Br > 11/4$, the internally generated heat by the viscous dissipation overcomes the heat supplied by the wall. When $Br \to +\infty$, Nu reaches an asymptotic value. When wall cooling ($Br < 0$) is applied to reduce the bulk temperature of the fluid, as explained earlier, the amount of viscous dissipation may change the overall heat balance. With increasing value of Br in negative direction, the Nusselt number reaches an asymptotic value. As shown, increasing the Knudsen number changes the location of the singularity point and the asymptotic values of Nu. The variation of the singular value of the Brinkman number, Br_s with the increasing Knudsen number is shown in Fig. 4b. Using the modified Brinkman number for the CHF case, the variation of Nu with Br_q for different

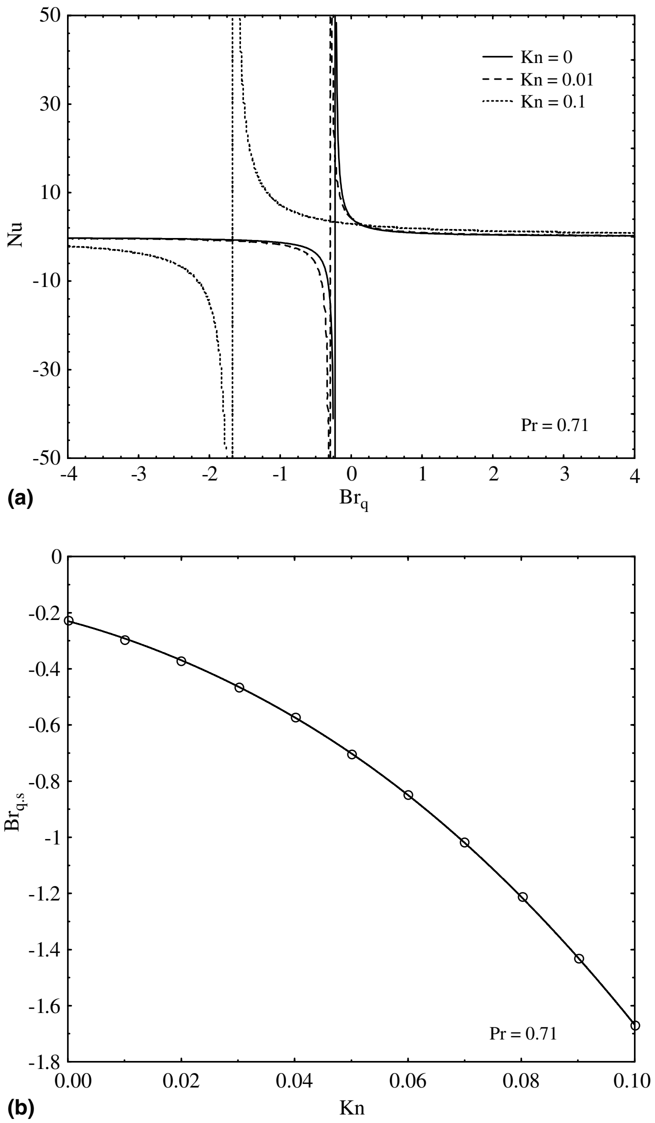

Fig. 5. (a) The influence of Br_q, on Nu at various Kn for the CHF case. (b) The variation of $Br_{q,\mathrm{s}}$ with Knudsen number for the CHF case.

Kn is shown in Fig. 5a. Again, singularities are observed. As noticed, when Br_q goes to infinity for either wall heating or wall cooling case, the Nusselt number reaches the same asymptotic value, $Nu = 0$. This is due to fact that the heat generated internally by viscous dissipation processes will balance the effect of wall heating or cooling, reaching a thermal equilibrium condition. Fig. 5b shows the variation of the singular value of the modified Brinkman number, $Br_{q,\mathrm{s}}$ with the increasing Knudsen number. As seen, opposite to Br_s, $Br_{q,\mathrm{s}}$ decreases with Kn.

For the CWT condition, Fig. 6a illustrates the variation of Nu with Br. The behavior observed can be explained similarly to that for the CHF condition. Again, singularities are observed. And, the variation of the singular value of the Brinkman number, Br_s with Kn is given in Fig. 6b. An increase at Kn results in increasing Br_s.

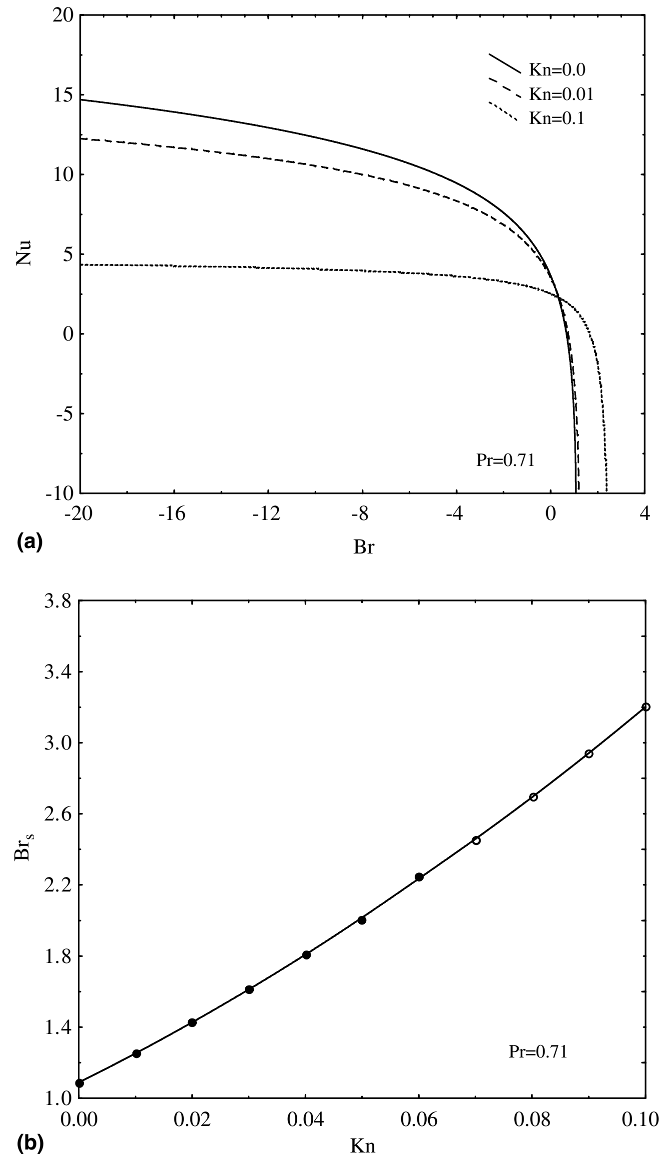

Fig. 6. (a) The influence of Br, on Nu at various Kn for the CWT case. (b) The variation of Br_s with Knudsen number for the CWT case.

4. Conclusions

The problem of both hydrodynamically and thermally fully developed forced convection in a micropipe has been studied. The slip condition and temperature jump at the wall, and the viscous dissipation are included in the analysis. The interactive effects of the Brinkman number and Knudsen number on the Nusselt number have been studied in detail. For the CHF case, $Nu = f(Br, Kn)$ and $Nu = f(Br_q, Kn)$ have been developed. The variation of the Nusselt number with the Brinkman or the modified Brinkman number presented some singularities. These singularities have been shown to originate from the thermal energy balance between the wall heat and the viscous dissipation heat during the thermal transport and the structure of the related formulations. For low values of the Br or Br_q, the Knudsen number has been shown to decrease the Nusselt number.

References

[1] M. Gad-el-Hak, Flow physics in MEMS, Mec. Ind. 2 (2001) 313–341.
[2] A. Beskok, G.E. Karniadakis, Simulation of heat and momentum transfer in complex micro-geometries, J. Thermophys. Heat Transfer 8 (1994) 355–370.
[3] C.M. Ho, C.Y. Tai, Micro-electro-mechanical systems (MEMS) and fluid flows, Annu. Rev. Fluid Mech. 30 (1998) 579–612.
[4] B. Palm, Heat transfer in microchannels, Microscale Thermophys. Eng. 5 (2001) 155–175.
[5] C.B. Sobhan, S.V. Garimella, A comparative analysis of studies on heat transfer and fluid flow in microchannels, Microscale Thermophys. Eng. 5 (2001) 293–311.
[6] N.T. Obot, Toward a better understanding of friction and heat/mass transfer in microchannels—a literature review, Microscale Thermophys. Eng. 6 (2002) 155–173.
[7] A.A. Rostami, N. Saniei, A.S. Mujumdar, Liquid flow and heat transfer in microchannels: a review, Heat Technol. 18 (2000) 59–68.
[8] A.A. Rostami, A.S. Mujumdar, N. Saniei, Flow and heat transfer for gas flowing in microchannels: a review, Heat Mass Transfer 38 (2002) 359–367.
[9] Z.Y Guo, Z.X. Li, Size effect on microscale single-phase flow and heat transfer, Int. J. Heat Mass Transfer 46 (2003) 59–149.
[10] Z.Y Guo, Z.X. Li, Size effect on single-phase channel flow and heat transfer at microscale, Int. J. Heat Fluid Flow 24 (3) (2003) 284–298.
[11] G.L. Morini, Single-phase convective heat transfer in microchannels: a review of experimental results, Int. J. Therm. Sci. 43 (2004) 631–651.
[12] P. Gravesen, J. Branebjerg, O.S. Jensen, Microfluidics—a review, J. Micromech. Microeng. 3 (1993) 168–182.
[13] J.C. Harley, Y. Huang, H.H. Bau, J.N. Zemel, Gas flow in microchannels, J. Fluid Mech. 285 (1995) 257–274.
[14] R.F Barron, X.M. Wang, R.O. Warrington, T. Amel, Evaluation of the eigenvalues for the Graetz problem in slip-flow, Int. Commun. Heat Mass Transfer 23 (4) (1996) 563–574.
[15] R.F. Barron, X. Wang, T.A. Ameel, R.O. Warrington, The Graetz problem extended to slip-flow, Int. J. Heat Mass Transfer 40 (8) (1997) 1817–1823.
[16] T.A. Ameel, X.M. Wang, R.F. Barron, R.O. Warrington, Laminar forced convection in a circular tube with constant heat flux and slip flow, Microscale Therm. Eng. 1 (4) (1997) 303–320.
[17] C.P. Tso, S.P. Mahulikar, Use of the Brinkman number for single phase forced convective heat transfer in microchannels, Int. J. Heat Mass Transfer 41 (1998) 1759–1769.
[18] C.P. Tso, S.P. Mahulikar, The role of the Brinkman number in analyzing flow transitions in microchannels, Int. J. Heat Mass Transfer 42 (1999) 1813–1833.
[19] C.P. Tso, S.P. Mahulikar, Experimental verification of the role of Brinkman number in microchannels using local parameters, Int. J. Heat Mass Transfer 43 (2000) 1837–1849.
[20] G. Tunc, Y. Bayazitoglu, Heat transfer in microtubes with viscous dissipation, Int. J. Heat Mass Transfer 44 (2001) 2395–2403.
[21] K.C. Toh, X.Y. Chen, J.C. Chai, Numerical computation of fluid flow and heat transfer in microchannels, Int. J. Heat Mass Transfer 45 (2002) 5133–5141.
[22] B. Xu, K.T. Ooi, C. Mavriplis, M.E. Zaghloul, Evaluation of viscous dissipation in liquid flow in microchannels, J. Micromech. Microeng. 13 (2003) 53–57.
[23] J. Koo, C. Kleinstreuer, Liquid flow in microchannels: experimental observations and computational analyses of microfluidics effects, J. Micromech. Microeng. 13 (2003) 568–579.
[24] J. Koo, C. Kleinstreuer, Viscous dissipation effects in microtubes and microchannels, Int. J. Heat Mass Transfer 47 (2004) 3159–3169.
[25] S.S. Hsieh, H.H. Tsai, C.Y. Lin, C.F. Huang, C.M. Chien, Gas flow in a long microchannel, Int. J. Heat Mass Transfer 47 (17–18) (2004) 3877–3887.
[26] O. Aydın, Effects of viscous dissipation on the heat transfer in a forced pipe flow. Part 1: both hydrodynamically and thermally fully developed flow, Energy Convers. Manage. 46 (2005) 757–769.
[27] L.S. Pan, T.Y. Ng, D. Xu, G.R. Liu, K.Y. Lam, Determination of temperature jump coefficient using the direct simulation Monte Carlo method, J. Micromech. Microeng. 12 (2002) 41–52.
[28] P.H. Oosthuizen, D. Naylor, Introduction to Convective Heat Transfer Analysis, McGraw-Hill, New York, 1999, pp. 158–167.